Homework 7

Linear Algebra 2
March 31, 2022

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Instructions Mention your name and *code name* on top of your solution sheet. Deadline for submitting this homework is 7$^{\text{th}}$ April 2022, 14:00 hrs.

Problem 1. *Decide and justify whether the following maps define an inner product*

(i) $\langle x, y\rangle = |x_1y_1| + |x_2y_2|$ *where* $x, y \in \mathbb{R}^2$

(ii) $\langle x, y\rangle = x_1y_1 + x_3y_3$ *where* $x, y \in \mathbb{R}^3$

Problem 2. *Prove or disprove the following the following statement:*
There is an inner product on $\mathbb{R}^2$ *such that the associated norm is given by* $\|(x, y)\| = \max\{x, y\}$ *for all* $(x, y) \in \mathbb{R}^2$.

Problem 3. *Let* V *be a inner product space*

(i) Show that $\langle x + y, x - y\rangle = \|x\|^2 - \|y\|^2$ *for every* $x, y \in V$.

(ii) Show that if x, y *have the same norm, then* $x + y$ *is orthogonal to* $x - y$.

(iii) Show that the diagonals of any rhombus are perpendicular to each other.

Problem 4. *Prove that for* $x, y, z, w \in \mathbb{R}_{>0}$ $(x + y + z + w)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right) \geq 16$.

Problem 5. *Let* $A \in \mathbb{R}^{n\times n}$ *be a matrix such that* $\|Av\| \leq \|v\|$ *for every* $v \in V$. *Show that* $A - \sqrt{3}I_n$ *is invertible.*