

Homework 8

Linear Algebra 2

April 7, 2022

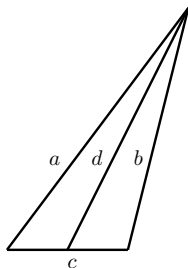
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Instructions Mention your name and *code name* on top of your solution sheet. Deadline for submitting this homework is 14th April 2022, 14:00 hrs. First question carries 4 points.

Problem 1. *Prove or disprove the following statement: There exists four points in the plane such that the pairwise distances between them are all odd integers.*

Hint: If n is an odd integer then $n^2 \equiv 1 \pmod{8}$

Problem 2. *In a triangle with length of the sides a , b , and c , let d be the length of the line segment from the midpoint of the side of length c to the opposite vertex. Then show that $a^2 + b^2 = \frac{1}{2}c^2 + 2d^2$.*



You studied this in high school as Apolloniuss Identity.

Hint: First prove that for any three vectors $u, v, w \in \mathbb{R}^n$ $\|w - \frac{1}{2}(u + v)\|^2 = \frac{\|w-u\|^2 + \|w-v\|^2}{2} - \frac{\|u-v\|^2}{4}$.

Problem 3. *Trace of a matrix is the sum of the elements on its principle diagonal, i.e., if $A = (a_{ij})_{n \times n}$ then $\text{trace}(A) = \sum_{i=1}^n a_{ii}$*

(i) *Show that $\langle A, B \rangle = \text{trace}(A^T B)$ defines an inner product on $\mathbb{R}^{n \times n}$.*

(ii) *Show that $\text{trace}(A)^2 \leq n \cdot \text{trace}(A^T A)$ for all $A \in \mathbb{R}^{n \times n}$,*

Problem 4. *Show that the square of an average is less than or equal to the average of the squares when the number are all real.*