Homework 9

Linear Algebra 2
April 21, 2022

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Instructions Mention your name and *code name* on top of your solution sheet. Deadline for submitting this homework is 28^{th} April 2022, 14:00 hrs.

Problem 1. *Let U be a finite-dimensional subspace of V. For $v \in V$, write $v = u + w$ where $u \in U$ and $w \in U^{\perp}$. The orthogonal projection of V onto U is a linear transformation defined as $P_U(v) = u$. Prove that $P_U P_W = 0$ if and only if $\langle u, w \rangle = 0$ for all $u \in U$ and $w \in W$, where U and W are finite-dimensional subspaces of V.*

Problem 2. *Find an orthonormal basis of a subspace of $\mathbb{R}^3$ defined as the solution space of the equation $x + y + z = 0$.*

Problem 3. *Consider the vector space $\mathbb{R}^4$ with the standard inner product. Using the Gram-Schmidt orthogonalization process, find an orthonormal basis of the row space of the following matrix*

$$\begin{pmatrix} 1 & 0 & 2 & 4 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 5 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

Problem 4. *Compute the distance of the point $A = [1, 2, 3, 4]$ from the plane passing through the origin and the points $B = [1, 1, 0, 0]$ and $C = [1, 1, 1, 2]$.*

Problem 5. *(i) Let U be a finite-dimensional subspace of V. Prove that $U^{\perp} = \{0\}$ if and only if $U = V$.*

(ii) Let $\mathcal{C}([-1, 1])$ be the vector space of continuous real-valued functions on the interval $[-1, 1]$ with inner product given by $\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)dx$ for $f, g \in \mathcal{C}([-1, 1])$. Let U be the subspace defined by $U = \{f \in \mathcal{C}([-1, 1]) \colon f(0) = 0\}$. Show that $U^{\perp} = \{0\}$.

Note: The second question tells us that the assumption U is finite-dimensional is necessary for the first question.